Distributed Registration of a Network of Asynchronous Sensors

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Abstract – *Registration of multiple sensors through common targets of opportunity is an extensively studied problem. The majority of proposed methods for computationally efficient estimation of sensor biases considered only the case of synchronous sensors. The relatively recent EXX method, however, allows exact estimation (under certain conditions) of sensor biases of asynchronous sensors. Unfortunately, the EXX method requires all measurements (or pseudomeasurements) originated by the targets of opportunity, which implies in high communication costs for large networks of sensors. In this paper, we formulate an extension of the EXX method that can be used for distributed bias estimation, i.e. obtains exact joint bias estimates for the entire network of sensors from joint bias estimates from subsets of these sensors. The proposed method can also be hierarchized in any manner, and can work with dissimilar sensors and different forms of sensor biases, thus being highly suited for today's demands of distributed data fusion.*

Keywords: Sensor bias estimation, registration, asynchronous sensors, distributed fusion.

1 Introduction

In multisensor data fusion, sensor registration, which involves estimation of systematic errors (biases) of a sensor, is a necessary step for effective alignment of data. In target tracking, for instance, if biases are not taken into account, data from different sensors but originated from the same target may not be identified as such, and tracking errors may be large.

Online bias estimation techniques typically involve executing the bias estimation together with the target tracking process. An optimal Bayesian approach to the problem would be to stack the states of all targets with all sensor biases and recursively estimate this augmented state. Such implementation would be computationally unfeasible in most situations, and prone to numerical problems for ill-conditioned systems.

While it has been shown that an augmented state system fed by measurements from a single sensor still has a completely observable state [1], most bias estimation methods use measurements originated from targets of opportunity commonly observed by two or more sensors; this approach allows decoupling of target state estimation from bias estimation, resulting in large reduction of computational complexity.

In this introduction we will recall some of these methods. Unfortunately, we couldn't find enough space to mention the broad range of works published in this area, so to be fair to their authors, we will focus only on papers that addressed the problem of bias estimation using asynchronous sensors, i.e. the case where measurements from different sensors are not time-coincident.

In [2], an one-step fixed-lag IMM predictor that translates track estimates to a common time is proposed to handle bias estimation using asynchronous sensors; this method, however, can only achieve relative registration under a relatively restrictive assumption that the distance between two sensors is small.

In [1], a version of the well-known two-stage Kalman filter ([3], [4]) for the case of asynchronous measurements is proposed. The method considers that the filter gain computed assuming bias-free measurements and the filter gain computed assuming measurements with known bias are identical. This actually holds when measurements are truly linear with respect to target states, but usually the measurement model must be linearized and filter gains are typically state-dependent (note that this also happens when measurements are converted to the target state coordinate system using "unbiased" conversions; see [5]). In these cases, the two-stage Kalman filter is suboptimal even assuming the validity of the linearized measurement model.

In [6], Lin et al. present the "EXX method" to allow estimation of biases from asynchronous sensors with phase difference but same sampling rate, by combining sets of three subsequent measurements (called proper time slots) into pseudomeasurements. Later in [7], this work was extended to asynchronous sensors with arbitrary sampling rates, by combining sets of measurements, each with cardinality $N > 3$, into $N - 2$ pseudomeasurements with correlated noise. The method ensures, however, that pseudomeasurements from differents proper time slots have independent noises. A relevant aspect of the EXX method is that it is exact conditioned on the pseudomeasurements and their underlying model (which is obtained from linearizations).

In [8], a variant bias estimator for asynchronous sensors is proposed, where a pseudomeasurement is generated for every subsequent measurement; since the pseudomeasurements have correlated noise, this method is clearly suboptimal and for that reason it is not considered on this paper. Finally, Qi et al. [9] extended the method proposed in [7] to allow bias estimation using targets of opportunity observed by an arbitrary (≥ 3) number of asynchronous sensors, by combining all measurements received in a time window into one pseudomeasurement. However, it requires the weighting cofficient system of equations to have a solution for the desired number of sensors, which is not guaranteed.

The works [6], [7], and [9] assume a single fusion center (FC) that processes measurements associated with targets of opportunity (or preprocessed measurements in the form of pseudomeasurements) to jointly estimate sensor biases. If the number of sensors is large, however, the communication cost associated with sending all measurements to a single FC may be prohibitive. Another limitation of the mentioned methods is that their formulation does not make explicit their adaption to non-radar sensors, and more importantly, to dissimilar sensors. It is also not clear on how to use these methods for other forms of sensor biases (besides range, azimuth and elevation offset and scale biases) or other target motion models (which are necessary to handle sensor asynchronicity).

This paper is best described as a direct extension of these works. First, we will generalize the formulation of the EXX method to any sensor, bias ou target model, and to an arbitrary (≥ 3) number of sensors. Second, we will derive a distributed bias estimator which is exact under the same conditions. The estimator is distributed in the sense that it is able to process local estimates of sensor biases (which, more specifically, consist of bias estimates based on a subset of measurements of the sensor suite). As in distributed target tracking, such local estimates may be transmitted at reduced rate, resulting in communication savings.

The paper is organized as follows: Section 2 describes the model and assumptions for bias estimation. Section 3 presents the derivation of the general distributed bias estimator proposed on this paper. In Section 4, simulation is used to illustrate the equivalence of the method to centralized bias estimation. Section 5 draws conclusions.

2 Bias model

In [6], [7] and related works, the same biased measurement model is used, with bias state composed of azimuth and range bias (both scale and offset biases), state in 2D cartesian coordinates, and measurements in polar coordinates converted to cartesian coordinates. In this paper, we will use a more general formulation that can be adapted to various sensor, bias and target models.

Let Ω be the coordinate system of target position (such as cartesian, ECEF, or stereographic) in $\mathbb{R}^n$, which will also be the registration coordinate system, and let Ξ be the coordinate system of measurements (such as polar, spherical, or cylindrical) in $\mathbb{R}^m$. Let $f_{\Omega\Xi} : \mathbb{R}^n \to \mathbb{R}^m$ be a function that converts a vector in Ω coordinates to Ξ coordinates, and let $f_{\Xi\Omega} : \mathbb{R}^m \to \mathbb{R}^n$ be the inverse function. If the inverse function does not exist, an approximation may be used, possibly using estimated target data.

Let $z_i(t_j)$ be the measurement generated by sensor i at time t_j. Then usually $z_i(t_j)$ can be written as

$$z_i(t_j) = f_{\Omega\Xi}(H(t_j)\mathbf{x}(t_j)) + c(\beta_i(t_j)) + w_i(t_j) \quad (1)$$

where $\mathbf{x}(t_j)$, $\beta_i(t_j)$ and $w(t_j)$ are respectively the true target state, bias state (with size p) and measurement noise (assumed zero-mean and white, with covariance $E[w_i(t_j)w_i(t_j)^T] = R_i(t_j)$), $H(t_j)$ is the observation matrix, and $c : \mathbb{R}^p \to \mathbb{R}^m$ is a known function such that $c(0_{p\times p}) = 0_{m\times m}$. The bias vector $\beta_i(t_j)$ may have any form; besides typical azimuth and range scale/offset errors, it may include axis tilt and antenna pitch, for instance. If we linearize (1) with respect to $\beta_i(t_j)$ (with linearization centered on $\beta_i(t_j) = 0_{p\times p}$), we obtain

$$z_i(t_j) \approx f_{\Omega\Xi}(H(t_j)\mathbf{x}(t_j)) + C_i(t_j)\beta_i(t_j) + w_i(t_j) \quad (2)$$

where $C_i(t_j)$ is the jacobian of transformation $c(.)$ evaluated at $\beta_i = 0_{p\times p}$. Conversion to Ω coordinates and a second linearization (now with respect to both $\beta_i(t_j)$ and $w_i(t_j)$) yields:

$$z_i^{\Omega}(t_j) \approx H(t_j)\mathbf{x}(t_j) + B_i(t_j)C_i(t_j)\beta_i(t_j) + \gamma_i(t_j) \quad (3)$$

where $z_i^{\Omega}(t_j) = f_{\Xi\Omega}(z_i(t_j))$, $B_i(t_j)$ is the jacobian of transformation $f_{\Xi\Omega}(.)$ evaluated at $\beta_i = 0_{p\times p}$ and $w_i(t_j) = 0_{m\times m}$, and $\tilde{R}_i(t_j) = E[\gamma_i(t_j)\gamma_i(t_j)^T] = B_i(t_j)R(t_j)B_i(t_j)^T$.

Note that (3) has the same form of the 2D radar bias model used in [6] and [7] to derive the EXX method. This shows that the method may actually be applied to any combination of measurement and registration coordinates, and keeps its exactness property as long as the transformation $f_{\Omega\Xi} : \mathbb{R}^n \to \mathbb{R}^m$ is invertible.

3 Registration of an arbitrary number of asynchronous sensors

3.1 Pseudomeasurement model

We will review and extend the pseudomeasurement model of the EXX method since it will be used in the derivation. The idea of the method is to combine subsequent measurements or tracks into pseudomeasurements, which can be used for estimation of sensor biases without dependence on true target position. A set of asynchronous measurements used to form pseudomeasurements is called a proper time slot. Note that in all derivations of this section, we will assume that the data association problem is resolved, i.e. that we know which measurements are originated by which targets. This is accomplished by having an external multi-sensor multi-target tracking process that feeds the bias estimator with target-associated measurements.

As described in [7], a proper time slot may be formed by three subsequent measurements (originated by the same target) from two sensors, or by n measurements ($n > 3$) where the first $n-1$ measurements are from the same sensor, and the last measurements are from another sensor. As we will see, a proper time slot may also be formed by three subsequent measurements from three different sensors. In this way, any sequence of measurements originated by a target, generated by any number of sensors, can be entirely partitioned into proper time slots, as shown on Figure 1.

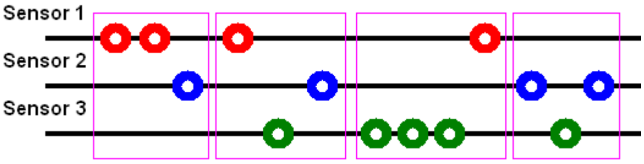

Figure 1: Proper time slots formed by asynchronous measurements

In [6] and [7], it is assumed that target motion is described by a two-dimensional Discretized Continuous White Noise Acceleration (DCWNA) model [10]. For the sake of generality, we will instead consider any motion model of the form

$$\mathbf{x}(t_j) = F(t_j, t_i)\mathbf{x}(t_i) + v(t_j, t_i) \tag{4}$$

where $F(t_j, t_i)$ is the transition matrix from time t_i to time t_j, and the process noise $v(t_j, t_i)$ is zero-mean, white with covariance $Q(t_j, t_i)$.

3.1.1 Proper time slot with three measurements

Consider a proper time slot with three measurements $z_{s_1}(t_1)$, $z_{s_2}(t_2)$ and $z_{s_3}(t_3)$ already converted to registration coordinates (with the superscript Ω omitted for simplicity). The subscripts s_1, s_2 and s_3 denote the sensor indexes, which form the set $S = \{s_1\} \cup \{s_2\} \cup \{s_3\}$ (this notation is used since two sensors indexes may be equal). In this case, the corresponding pseudomeasurement is

$$z(t_2) = z_{s_2}(t_2) - [\alpha_1 z_{s_1}(t_1) + \alpha_2 z_{s_3}(t_3)] \tag{5}$$

where the weighting constants α_1 and α_2 are the solutions of system

$$H(t_2) - \alpha_1 H(t_1) F(t_2, t_1)^{-1} - \alpha_2 H(t_3) F(t_3, t_2) = 0_{m\times n} \tag{6}$$

Note that system (6) may not necessarily have a solution. However, in this generalization we require that only this system has a solution, while in [9] it's required that the equivalent system with $r-1$ variables (where r is the number of sensors) has a solution. For the particular case of target motion described by the DCWNA model in all dimensions, there is a single solution given by [6]

$$\alpha_1 = \frac{t_3 - t_2}{t_3 - t_1}$$
$$\alpha_2 = \frac{t_2 - t_1}{t_3 - t_1}.$$

Now, considering that each sensor $i \in S$ has a (possibly time-varying[1]) bias β_i, using (3), (4) and (6), pseudomeasurement expression (5) becomes:

$$\begin{aligned} z(t_2) = {} & B_{s_2}(t_2)C_{s_2}(t_2)\beta_{s_2} - \alpha_1 B_{s_1}(t_1)C_{s_1}(t_1)\beta_{s_1} \\ & - \alpha_2 B_{s_3}(t_3)C_{s_3}(t_3)\beta_{s_3} - \alpha_1 A_0 v(t_2, t_1) \\ & - \alpha_2 H(t_2) v(t_3, t_2) + \gamma_{s_2}(t_2) - \alpha_1 \gamma_{s_1}(t_1) \\ & - \alpha_2 \gamma_{s_3}(t_3) \end{aligned} \tag{7}$$

where

$$A_0 = -H(t_1)F(t_2, t_1)^{-1}. \tag{8}$$

Finally, by rearranging the terms of (7), we obtain the expression:

$$z(t_2) = \mathcal{H}(t_2)\beta_S + \tilde{w}(t_2) \tag{9}$$

where $\beta_S = \begin{bmatrix}\beta_{s_1}^T & \beta_{s_2}^T & \beta_{s_3}^T\end{bmatrix}^T$, and $\mathcal{H}(t_2)$ and $\tilde{w}(t_2)$ are given by

$$\begin{aligned} \mathcal{H}(t_2) = {} & [-\alpha_1 B_{s_1}(t_1)C_{s_1}(t_1), \\ & B_{s_2}(t_2)C_{s_2}(t_2), \\ & - \alpha_2 B_{s_3}(t_3)C_{s_3}(t_3)] \end{aligned} \tag{10}$$

$$\begin{aligned} \tilde{w}(t_2) = {} & -\alpha_1 A_0 v(t_2, t_1) - \alpha_2 H(t_2) v(t_3, t_2) \\ & + \gamma_{s_2}(t_2) - \alpha_1\gamma_{s_1}(t_1) - \alpha_2\gamma_{s_3}(t_3). \end{aligned} \tag{11}$$

Naturally, if S is composed by only two sensors, we may eliminate the redundant component of β_S and sum the blocks of $\mathcal{H}(t_2)$ referring to the same sensor. We can also see that $E[\tilde{w}(t_2)\tilde{w}(t_i)^T] = \delta(2-i)\mathcal{R}(t_2)$ for $i \neq 1$ and $i \neq 3$, with

$$\begin{aligned} \mathcal{R}(t_2) = {} & \tilde{R}_{s_2}(t_2) + \alpha_1^2 \tilde{R}_{s_1}(t_1) + \alpha_2^2 \tilde{R}_{s_3}(t_3) \\ & + \alpha_1^2 A_0 Q(t_2, t_1) A_0^T + \alpha_2^2 H(t_3) Q(t_3, t_2) H(t_3)^T. \end{aligned} \tag{12}$$

Since the pseudomeasurement model is linear and pseudomeasurements are independent conditioned on bias state, if the targets are constantly observed by the same of set of sensors, then β_S can be optimally estimated using a recursive LS estimator (for static bias) or a Kalman filter (for time-varying bias). This is the exactness property of the EXX method.

It should be noted however that optimality of bias estimation conditioned on pseudomeasurements does not imply optimality conditioned on actual measurements of the system, since the true measurements cannot be uniquely determined from pseudomeasurements, and the pseudomeasurement model is based on linearization of the true measurement model (as remarked by [7], however, the effect of linearization errors can be mitigated through use of unbiased coordinate conversions such as [11],[12]).

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[1] The time index for time-varying bias will be omitted for convenience, unless necessary due to context.

3.1.2 Proper time slot with 4+ measurements

Consider now a proper time slots with $n > 3$ measurements $z_1(t_1)$, $z_1(t_2)$ ... $z_1(t_{n-1})$ and $z_2(t_n)$ (we use 1 and 2 instead of s_1 and s_2 since here the sensor indexes are necessarily distinct).

First, for $j \geq 2$, we rewrite (4) as

$$\mathbf{x}(t_j) = F(t_j, t_2)\mathbf{x}(t_2) + \sum_{k=3}^{j} F(t_j, t_k) v(t_k, t_{k-1}). \tag{13}$$

The proper time slot yields $n - 2$ pseudomeasurements, each with time index j taken from $1 < j < n$. The pseudomeasurement equation for index j is

$$z(t_j) = z_2(t_n) - [\alpha_1 z_1(t_1) + \alpha_2 z_1(t_j)] \tag{14}$$

where the weighting constants α_1 and α_2 are the solutions of system

$$H(t_n) - \alpha_1 H(t_1) F(t_2, t_1)^{-1} - \alpha_2 H(t_j) F(t_j, t_2) = 0_{m \times n}. \tag{15}$$

Note that α_1 and α_2 are both functions of time index j, which will be usually omitted, unless necessary due to context. For the DCWNA model, the solution is [7]

$$\begin{aligned} \alpha_1 &= -\frac{t_n - t_j}{t_j - t_1} \\ \alpha_2 &= \frac{t_n - t_1}{t_j - t_1} \end{aligned}$$

Using (3), (13) and (15), pseudomeasurement expression (14) becomes

$$\begin{aligned} z(t_j) = {} & B_2(t_n) C_2(t_n) \beta_2 - \alpha_1 B_1(t_1) C_1(t_1) \beta_1 \\ & - \alpha_2 B_1(t_j) C_1(t_j) \beta_1 - \alpha_1 A_0 v(t_2, t_1) \\ & + \sum_{k=3}^{j} A_1(j, k) v(t_k, t_{k-1}) \\ & + \sum_{k=j+1}^{n} A(n, k) v(t_k, t_{k-1}) + \gamma_2(t_n) \\ & - \alpha_1 \gamma_1(t_1) - \alpha_2 \gamma_1(t_j) \end{aligned} \tag{16}$$

where

$$\begin{aligned} A_0 &= -H(t_1) F(t_2, t_1)^{-1} \\ A(j, i) &= H(j) F(t_j, t_i) \\ A_1(j, i) &= A(n, i) - \alpha_2 A(j, i). \end{aligned}$$

Rearranging the terms of (16), we obtain the expression:

$$z(t_j) = \mathcal{H}(t_j) \beta_S + \tilde{w}(t_j) \tag{17}$$

where $\beta_S = \begin{bmatrix} \beta_1^T & \beta_2^T \end{bmatrix}^T$, and $\mathcal{H}(t_j)$ and $\tilde{w}(t_j)$ are given by

$$\begin{aligned} \mathcal{H}(t_j) = {} & [-\alpha_1 B_1(t_1) C_1(t_1) - \alpha_2 B_1(t_j) C_1(t_j), \\ & B_2(t_n) C_2(t_n)] \end{aligned} \tag{18}$$

$$\begin{aligned} \tilde{w}(t_j) = {} & -\alpha_1 A_0 v(t_2, t_1) + \sum_{k=3}^{j} A_1(j, k) v(t_k, t_{k-1}) \\ & + \sum_{k=j+1}^{n} A(n, k) v(t_k, t_{k-1}) + \gamma_2(t_n) \\ & - \alpha_1 \gamma_1(t_1) - \alpha_2 \gamma_1(t_j). \end{aligned} \tag{19}$$

Now, we verify that, for $1 < j < n$:

$$E[\tilde{w}(t_j) \tilde{w}(t_i)^T] = \begin{cases} \mathcal{R}(j), & i = j \\ \mathcal{R}(j, i), & i \neq j \text{ and } 1 < i < n \\ 0_{m \times m}, & i < 1 \text{ or } i > n \end{cases} \tag{20}$$

where, assuming $i < j$ without loss of generality

$$\begin{aligned} \mathcal{R}(j) = {} & \tilde{R}_2(t_n) + \alpha_1^2 \tilde{R}_1(t_1) + \alpha_2^2 \tilde{R}_1(t_j) \\ & + \alpha_1^2 A_0 Q(t_2, t_1) A_0^T \\ & + \sum_{k=3}^{j} A_1(j, k) Q(t_k, t_{k-1}) A_1(j, k)^T \\ & + \sum_{k=j+1}^{n} A(n, k) Q(t_k, t_{k-1}) A(n, k)^T \end{aligned} \tag{21}$$

$$\begin{aligned} \mathcal{R}(j, i) = {} & \tilde{R}_2(t_n) + \alpha_1(j) \tilde{R}_1(t_1) \alpha_1(i) \\ & + \alpha_1(j) A_0 Q(t_2, t_1) A_0^T \alpha_1(i) \\ & + \sum_{k=3}^{i} A_1(j, k) Q(t_k, t_{k-1}) A_1(i, k)^T \\ & + \sum_{k=i+1}^{j} A_1(j, k) Q(t_k, t_{k-1}) A(n, k)^T \\ & + \sum_{k=j+1}^{n} A(n, k) Q(t_k, t_{k-1}) A(n, k)^T. \end{aligned} \tag{22}$$

Since the $n - 2$ pseudomeasurements yielded by the proper time slot are correlated conditioned on the bias state, application of a LS recusive estimator or a Kalman filter on (17) requires a batch update, i.e., the pseudomeasurements must be concatenated into a single pseudomeasurement vector. This can be computationally expensive since it requires the inversion of the concatenated pseudomeasurement noise matrix, which is not block-diagonal. A suboptimal, but significantly simpler solution is to just discard the first measurement whenever a sequence of three measurements from the same sensor is found, which ensures that only proper time slots with three measurements are formed.

3.2 Centralized joint bias estimation

Let's assume that we have a set S of r asynchronous sensors, where each sensor $i \in S$ has a (possibly time-varying) bias β_i. Suppose also that we have M (possibly overlapping) subsets S_k of S, each of size r_k, such that $\bigcup_{k=1}^{M} S_k = S$.

Consider now a group of targets that generates a sequence of pseudomeasurements according to (9) and (17). Each pseudomeasurement is formed by measurements generated by a subset of sensors from S. Therefore, for each subset of sensors S_k, we will have a set of N_k pseudomeasurements $Z_k^{N_k} = \{z(t_j)\}_{j=1}^{N_k}$, with each having the form

$$z(t_j) = \mathcal{H}(t_j)\beta_{S_k} + \tilde{w}(t_j) \tag{23}$$

Our goal is to find the MMSE estimate $\hat{\beta}_S = E[\beta_S|Z^N]$ where $\beta_S = \begin{bmatrix}\beta_1^T & \beta_2^T & \dots & \beta_r^T\end{bmatrix}^T$ and Z^N denotes the set of all N pseudomeasurements generated by S. If we have a single fusion center (FC) which receives measurements associated with targets of opportunity from all sensors in S (or, alternatively, their equivalent pseudomeasurements), then the task is straightforward. We only need to extend the bias vector β_{S_k} to include the bias vectors from all sensors in S, and complete the corresponding observation matrix $\mathcal{H}(t_j)$ with zero block matrices. Then the estimate can be obtained using a recursive LS estimator or a Kalman filter, as in the case where the targets of opportunity are observed by a fixed set of sensors.

3.3 Distributed joint bias estimation

Let's now consider the same sensor suite S of the centralized case. We make the assumptions that each sensor $i \in S$ has *a priori* bias distribution given by $p(i) = \mathcal{N}(m_{i|0}, P_{i|0})$ (the notation $|0$ reminds that *a priori* distribution is being considered), and that sensor biases are independent, which means that a subset S_k of S will have *a priori* joint bias distribution given by $p(S_k) = \mathcal{N}(m_{S_k|0}, P_{S_k|0})$, where $P_{S_k|0}$ is a block-diagonal matrix. Note that we can treat a sensor's bias distribution as diffuse by making $P_{i|0}^{-1} \to 0_{p\times p}$ (this can be processed by performing estimation in information form, as it will be seen later).

However, we assume now that, due to communication contraints, it's not possible to send all measurements or pseudomeasurements from the sensor suite to a single FC. However, it's possible to send partial lists of measurements (or their equivalent pseudomeasurements) to a number of "local" bias estimators (LBEs) k, $1 \leq k \leq M$, each processing the set of pseudomeasurements $Z_k^{N_k}$, which in turn is originated by the subset of sensors $S_k = \{l_1, l_2, \dots, l_{r_k}\}$.

Consider that each LBE, using the centralized EXX method, is able to compute the moments of the (assumed gaussian) density $p(S_k|Z_k^{N_k}) = \mathcal{N}(m_{S_k|k}, P_{S_k|k})$[2] (here we use the notation $|k$ to indicate that the p.d.f is conditioned on set $Z_k^{N_k}$).

Let $\bar{S}_k$ be the complement of S_k in S, and without loss of generality, let's assume that β_S has the form $\beta_S = [\beta_{S_k}^T \;\; \beta_{\bar{S}_k}^T]^T$. Given our assumptions, $p(\beta_S|Z_k^{N_k}) = p(\beta_{S_k}|Z_k^{N_k})p(\beta_{\bar{S}_k})$, i.e. it is gaussian with moments given by

$$m_{S|k} = \begin{bmatrix} m_{S_k|k} \\ m_{\bar{S}_k|0} \end{bmatrix} \tag{24}$$

$$P_{S|k} = \begin{bmatrix} P_{S_k|k} & \mathbf{0} \\ \mathbf{0} & P_{\bar{S}_k|0} \end{bmatrix} \tag{25}$$

where $\mathbf{0}$ is a zero matrix.

If β_S does not have the form $\beta_S = [\beta_{S_k}^T \;\; \beta_{\bar{S}_k}^T]^T$, we only need to change the order of the corresponding elements in (24) and (25).

Our goal is to design a FC that is able to obtain estimate $\hat{\beta}_S = m_S$ (conditioned on Z^N) using only local estimates $\hat{\beta}_{S_k} = m_{S_k|k}$. We will provide the derivation based on the following conditions:

1. Sensor biases are static. If that's not the case, it's necessary to take the cross-correlation between local estimates in account, as on distributed target tracking [13].

2. No two pseudomeasurements processed by different LBEs may be formed by a common measurement (i.e. the partial lists of measurements sent to each LBE must be disjoint).

3. If a pseudomeasurement is originated by target trajectory $[i, j]_t$, where i is the initial time index, j is the final time index and t is the target index, then a pseudomeasurement from another LBE, originated by trajectory $[i', j']_{t'}$ must have either $t' \neq t$, $j' < i$ or $i' > j$ (i.e. the trajectory segments which originate the partial lists of measurements sent to each LBE must be disjoint).

A conservative way (but not the only way) of enforcing conditions 2 and 3 is to require LBEs to process pseudomeasurements from mutually exclusive geographic areas.

With all three conditions satisfied, the probability density function of β_S conditioned on Z^N is given by:

$$\begin{aligned} p(\beta_S|Z_N) &\propto p(Z_N|\beta_S)p(\beta_S) \\ &\propto \left(\prod_{k=1}^{M} p(Z_k^{N_k}|\beta_S)\right) p(\beta_S) \\ &\propto \frac{1}{p(\beta_S)^{M-1}} \left(\prod_{k=1}^{M} p(\beta_S|Z_k^{N_k})\right) \end{aligned} \tag{26}$$

where we use the fact that pseudomeasurements are independent conditioned on sensor biases. We recall that $p(\beta_S) = \mathcal{N}(m_{S|0}, P_{S|0})$ and $p(\beta_S|Z_k^{N_k}) = N(m_{S|k}, P_{S|k})$, and observe the following properties of multivariate normal distribution:

$$\prod_{k=1}^{M} \mathcal{N}(m_k, P_k) \propto \mathcal{N}(m_p, P_p) \tag{27}$$

$$\frac{\mathcal{N}(m_1, P_1)}{\mathcal{N}(m_2, P_2)} \propto \mathcal{N}(m_d, P_d) \tag{28}$$

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[2] Note that if we use a RLS estimator with the EXX, this will only hold if the RLS is mathematically equivalent to the MMSE estimator.

where

$$P_p = \left(\sum_{k=1}^{M} P_k^{-1}\right)^{-1}, \qquad m_p = P_p \sum_{k=1}^{M} P_k^{-1} m_k$$
$$P_d = \left(P_1^{-1} - P_2^{-1}\right)^{-1}, \quad m_d = P_d \left(P_1^{-1} m_1 - P_2^{-1} m_2\right).$$

Then, we have

$$p(\beta_S|Z_N) = \mathcal{N}(m_S, P_S) \tag{29}$$

where

$$P_S = \left(\sum_{k=1}^{M} P_{S|k}^{-1} - (M-1)P_{S|0}^{-1}\right)^{-1} \tag{30}$$
$$m_S = P_S \left(\sum_{k=1}^{M} (P_{S|k}^{-1} m_{S|k}) - (M-1)P_{S|0}^{-1} m_{S|0}\right). \tag{31}$$

3.3.1 Implementation in information form

Since there is a large number of matrix inversions in expression (29), it's convenient to perform mathematical manipulations with state vectors and covariance matrixes in information form. A mean vector m and the corresponding covariance matrix P are, in information form, respectively given by $\mathfrak{m} = P^{-1}m$ and $\mathfrak{P} = P^{-1}$.

In each LBE, the local estimates may be directly computed in information form. Let's recall the pseudomeasurement model given by (9). Since we assumed biases are static, $\{\mathfrak{P}_{S_k|k}\}$ and $\{\mathfrak{m}_{S_k|k}\}$ can be computed using the information form of the recursive LS estimator:

$$\mathfrak{P}_{S_k|k}(0) = \begin{bmatrix} \mathfrak{P}_{l_1|0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathfrak{P}_{l_2|0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathfrak{P}_{l_{r_k}|0} \end{bmatrix} \tag{32}$$
$$\mathfrak{m}_{S_k|k}(0) = \begin{bmatrix} \mathfrak{m}_{l_1|0} \\ \mathfrak{m}_{l_2|0} \\ \vdots \\ \mathfrak{m}_{l_{r_k}|0} \end{bmatrix} \tag{33}$$
$$\mathfrak{P}_{S_k|k}(t_j) = \mathfrak{P}_{S_k|k}(j-1) + \mathcal{H}(t_j)^T \mathcal{R}(t_j)^{-1} \mathcal{H}(t_j) \tag{34}$$
$$\mathfrak{m}_{S_k|k}(t_j) = \mathfrak{m}_{S_k|k}(j-1) + \mathcal{H}(t_j)^T \mathcal{R}(t_j)^{-1} z(t_j). \tag{35}$$

An obvious advantage of LS estimation in information form is that we can use diffuse *a priori* distributions for sensor biases, by setting $\mathfrak{P}_{i|0} = 0_{p\times p}$.

If we need to estimate time-varying biases (and the cross-correlation problem is either treated or ignored), we may use the information filter instead. The prediction step of the filter may be computationally expensive for large bias vectors or targets of opportunity observed by many sensors. If biases don't vary too rapidly in time, it is probably best to perform an information prediction only at significant time intervals.

To implement the FC, we first note that expressions (30) and (31) are equivalent to:

$$P_S = \left(\sum_{k=1}^{M} \mathfrak{P}_{S|k} - (M-1)\mathfrak{P}_{S|0}\right)^{-1} \tag{36}$$
$$m_S = P_S \left(\sum_{k=1}^{M} \mathfrak{m}_{S|k} - (M-1)\mathfrak{m}_{S|0}\right). \tag{37}$$

These expressions are even more simplified if P_S and m_S are expressed in information form, which is useful if the results of fusion are going to be transmitted to another FC (i.e. if we have an additional level of hierarchization). Obtaining $\mathfrak{P}_{S|0}$ and $\mathfrak{m}_{S|0}$ is straightforward since $P_{S|0}$ is block-diagonal. $\mathfrak{P}_{S|k}$ and $\mathfrak{m}_{S|k}$ may be easily obtained from $\mathfrak{P}_{S_k|k}$ and $\mathfrak{m}_{S_k|k}$. First, as previously done, we assume that $\beta_S = [\beta_{S_k}^T \;\; \beta_{\bar{S}_k}^T]^T$; then we have:

$$\mathfrak{m}_{S|k} = \begin{bmatrix} \mathfrak{m}_{S_k|k} \\ \mathfrak{m}_{\bar{S}_k|0} \end{bmatrix} \tag{38}$$
$$\mathfrak{P}_{S|k} = \begin{bmatrix} \mathfrak{P}_{S_k|k} & \mathbf{0} \\ \mathbf{0} & \mathfrak{P}_{\bar{S}_k|0} \end{bmatrix}. \tag{39}$$

If β_S does not have the form $\beta_S = [\beta_{S_k}^T \;\; \beta_{\bar{S}_k}^T]^T$, we only need to change the order of elements in (38) and (39), and we will verify that $\mathfrak{P}_{S|k}$ is a block matrix $[\mathfrak{P}_{S|k,ij}]_{r\times r}$ where

$$\mathfrak{P}_{S|k,ij} = \begin{cases} \mathfrak{P}_{S_k|k,i'j'}, & i,j \in S_k \\ \mathfrak{P}_{\bar{S}_k|0,i'i'}, & i = j \text{ and } i \notin S_k \\ 0_{p\times p}, & \text{otherwise} \end{cases} \tag{40}$$

where i' and j' are the indexes in $m_{S_k|k}$ or $m_{S_k|0}$ corresponding to the sensors with index i and j.

Similarly, $\mathfrak{m}_{S|k} = [\mathfrak{m}_{S|k,i}]_r$ where

$$\mathfrak{m}_{S|k,i} = \begin{cases} \mathfrak{m}_{S_k|k,i'}, & i \in S_k \\ \mathfrak{m}_{\bar{S}_k|0,i'}, & i \notin S_k \end{cases} \tag{41}$$

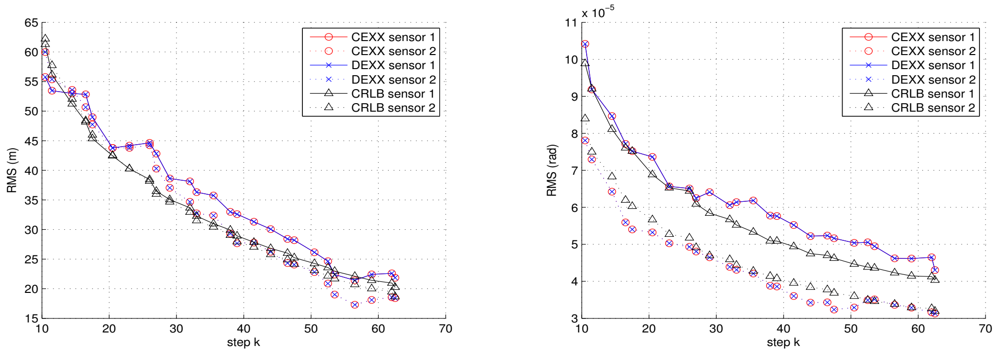
3.3.2 Properties

The derived distributed EXX method has the following nice properties:

- It can be hierarchized in any number of levels. This can be seen by looking at (26), where we verify that the FC may fuse estimates from other FCs instead of LBEs, as long as there is no loops in the fusion topology.

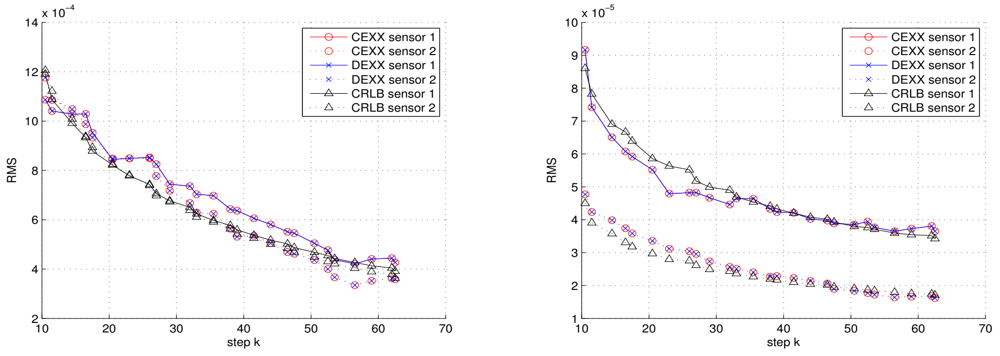
- It can process data from dissimilar sensors, by setting the apropriate $B_i(t_j)$ matrix for sensor i on (7) and (16).

- It is completely equivalent to the centralized EXX, even when the local estimates are received at reduced rate. As can be seen on (36) and (37), that is because the fused estimate depends only on the current local estimates.



(a) RMS errors CRLB of range offset biases

(b) RMS errors and CRLB of azimuth offset biases



(c) RMS errors and CRLB of range scale biases

(d) RMS errors and CRLB of azimuth scale biases

Figure 2: RMS errors and CRLB for EXX and EXX+ methods

4 Simulation

Consider the scenario with four radars shown on Figure 3, where the lines contain the initial position of targets (there is a total of 112 targets). All radars share the following properties: period $T = 2$ s, range noise standard deviation $\sigma_r = 10$ m, azimuth noise standard deviation $\sigma_\theta = 1$ mrad, and bias vector $\beta_1 = \ldots = \beta_4 = [100 \text{ m} \;\; 2 \text{ mrad} \;\; 0 \;\; 0]^T$, which is composed respectively of range offset bias, azimuth offset bias, range scale bias and azimuth scale bias. The radars are asynchronous, with offset from initial time $t = 0$ respectively given by $\Delta t_1 = 0$ s, $\Delta t_2 = 0.5$ s, $\Delta t_3 = 1$ s and $\Delta t_4 = 1.5$ s.

The dynamics of the targets are modeled using the DCWNA model, where the power spectral densities of the process noise covariance are given by $\tilde{q}_x = \tilde{q}_y = 6 \text{ m}^2/\text{s}^3$. The initial bias estimate of sensor i is given by $m_{i|0} = [0 \;\; 0 \;\; 0 \;\; 0]^T$ and the initial bias covariance is given by $P_{i|0} = \text{diag}[(500 \text{ m})^2 \;\; (200 \text{ mrad})^2 \;\; (10^{-2})^2 \;\; (10^{-3})^2]$.

We simulated both the centralized EXX (CEXX) and the distributed (DEXX) to check whether they are truly equivalent. Simulation was performed with 100 Monte Carlo runs, where on each run, the CEXX processed measurements generated by all radars, and the DEXX processed local bias estimates computed by four LBEs, with each LBE processing measurements generated on one of the four intersection areas from Figure 3. Since the solution computed by the CEXX is exact, we can evaluate the Cramer-Rao Lower Bound (CRLB) on the covariance of bias estimates computed using this method.

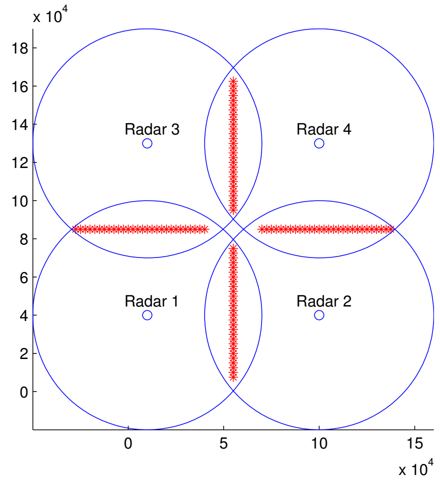


Figure 3: Simulation scenario

Figure 2 show the RMS errors and CRLB of the estimated bias state vector for radars 1 and 2. As expected, the DEXX and CEXX yield the same results (since they are mathematically equivalent), and the estimation errors yielded by both methods are consistent with the CRLB.

5 Conclusions

In this paper we provided an exact solution for distributed bias estimation of asynchronous sensors. The term "exact" should be used with care, since it is not exact conditioned on actual measurements of the system, but on pseudomeasurements which are derived from approximations. Besides, if the conversion from registration coordinates to measurement coordinates is not invertible, an approximate inverse conversion needs to provided, which may further degrade performance of estimation. Finding suitable approximations for typical registration schemes (such as registration of bearing-only sensors in 2D or 3D space, or registration of 2D radars in 3D space) is a challenge that needs to be addressed to make the method effective on practical situations.

The method is highly flexible and suited to various demands of distributed data fusion, such as multiple level hierarchization and data processing from dissimilar sensors. It still has, however, two important limitations. The first is that its exactness property does not apply to time-varying biases, although we assume that this can be addressed by applying cross-correlation estimation techniques used on distributed target tracking such as [13], [14] and [15].

The second, and probably more difficult to handle, is that the method requires some strict conditions on how measurements should be distributed to the LBEs, and it also does not allow loops in the distributed fusion topology. Possible extensions of this work may consider the use of diffusion least-mean squares estimators [16] to handle non-hierarchical and adaptative distributed fusion topologies.

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